

NUMERIC EVALUATION FROM CANTILEVER BEAM AND PLATE

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Abstract

As soon as fluid passes over a solid surface vortices are formed at the surface of structure to satisfy the no slip boundary conditions at the fluid-structure interface. Vortices that are formed at fluid-structure interface diffuse in fluid domain. In the present study, the strength of vortices is evaluated on the surface of an oscillating structural element (cantilever beam and plate) that is interacting with oscillating fluid flow. In the analysis, the analysis of structural element is carried out by finite element method and analysis of fluid domain is carried out by panel method. It is assumed that the deformed geometry of the structure at any moment is same shape as the vortex sheet. The structure is replaced by a virtual vortex sheet of uniformly distributed point vortices on the surface. A few numerical examples are presented to show the variation of vortex strength, lift and pressure coefficients for different type flow passes over the oscillating rigid and flexible structural element in fluid medium. The strength of vortex depends on fluid flow characteristics around the structural element and motion of structure in fluid medium.

Keywords: Vortices, solid-fluid interface, vortex sheet, vortex strength, structural element

1 Introduction

The process of vortex shedding is a classical problem in fluid mechanics and there have been many theoretical and experimental studies on various aspect of this problem. Vortex shedding from a bluff body has been investigated for many decades due to its important role in understanding fundamental fluid dynamics and its close link to field applications. Under particular conditions, the motion of

a fluid past an object results in the generation of a wake containing vortices in the trailing edge. In a flowing fluid, vortices are shed alternately from either side of the body, and the resulting changes in circulation around the body lead to fluctuating forces. The organized and periodic shedding of vortices may result in considerable fluctuating loading on the structure. As the frequency of shedding is approximately equal to one of the natural frequencies of structure, the structure often vibrates with large amplitude in a plane perpendicular to the flow direction. On the other hand when the structure is periodically oscillated by external forcing, the shedding frequency may be modified or shift from its natural shedding frequency to the forcing frequency.

Howe (1987) examined the surface pressure fluctuations due to periodic vortex shedding from the blunt trailing edge of a coated airfoil. Guocan and Caimao (1991) studied numerically on near wake flows of a flat plate and calculate the forces on a plate in steady, oscillatory and combined flows by using the discrete vortex model and improved vorticity creation method. Cortelezzi and Leonard (1993) considered a two-dimensional unsteady flow past a semi-infinite plate with transverse motion. The rolling-up of the separated shear-layer was modeled by a point vortex whose time dependent circulation was predicted by an unsteady Kutta condition. Ting and Perlin (1995) experimentally determined a boundary condition model for the contact line in oscillatory flow, for an upright plate, oscillated vertically with sinusoidal motion in dye laden water. Larsen and Walther (1997) simulated the two dimensional viscous incompressible flow past a flat plate of finite thickness and length using the discrete vortex method. Both a fixed plate and a plate undergoing a harmonic heave and pitch motion were studied. Lewandowski (2002) studied the non-linear vibrations of beams excited by vortex shedding.

The steady state responses of beams near the synchronization region were taken into account. The main aerodynamic properties of wind were described by using the semi empirical model proposed by Hartlen and Currie. The finite element method and strip method were used to formulate the equation of motion of the system treated. Qiu and Hsiung (2002) developed a panel-free method (PFM) based on the desingularized Green's formula, to solve the radiation problem of a

floating body in time domain. The velocity potential due to a non-impulsive velocity was obtained by solving the boundary integral equation in terms of source strength distribution. Khatir (2004) presented an approach for solving the source /sink boundary integral equation by using an indirect Boundary Element Method. The author implemented this analysis for solving flow over 3-D obstacles to impose impermeability at the wall in conjunction with discrete vortex method. Lin Lin, Ho, Chang, Hsieh, and Chang (2005) conducted experimental study on the vortex shedding process induced by the interaction between a solitary wave and a submerged vertical plate.

Particle image velocimetry (PIV) was used for quantitative velocity measurement while a particle tracing technique was used for qualitative flow visualization. Vortices were generated at the tip of each side of the plate. Tang and Pai (2007) developed a fluid-elastic model to study the dynamics of cantilevered plates in axial flow, with an additional support at the plate at trailing edge. A non-linear equation of motion, based on the inextensibility condition was used to account for the possible large vibration amplitude of the plate. For the fluid-dynamic part, an unsteady lumped vortex model was used for calculating the fluid loads acting on the plate undergoing deformation and for calculating the pressure difference across the plate and undulating wake streets behind the plate.

In the present study, the strength of vortices on the surface of a cantilever beam and plate is determined in fluid medium. The lift and pressure coefficients are evaluated in the structural element for sinusoidal fluid flow passes over the structural element. Initially the structure is assumed as rigid one subsequently the structure is considered as flexible. For determining the strength of vortices on the surface of structural element, the structure is replaced by vortex sheet and shape of the vortex sheet is assumed similar as the deformed shape of the structural element at any instant of time.

2 Theoretical Formulation

For weakly viscous fluid the boundary layer is very thin and the flow around the structure may be modeled by potential flow. If the fluid is assumed to be inviscid then there will be only tangential component of velocity at the solid

surface and normal component of velocity on the surface will be zero. The boundary layers on the solid surface and the wake of the trailing edge may be assumed as a thin layer of concentrated vorticity layer. There is a kinematic relationship between vorticity and velocity and it is valid for viscous as well inviscid fluid. In the real flow, the vorticity is such that the velocity associated with it satisfies the no penetration boundary conditions at the solid surface. Velocity induced at a point \vec{r}_1 due to vorticity $\vec{\Omega}$ located at \vec{r}_0 may be obtained by Biot-Savart law.

$$\vec{V}(\vec{r}_1) = \frac{1}{4\pi} \int_{\forall} \frac{\vec{\Omega} \times (\vec{r}_0 - \vec{r}_1)}{|\vec{r}_0 - \vec{r}_1|^3} d\forall \tag{1}$$

The induced velocity $v(\vec{r})$ at a point \vec{r} due to a point vortex of strength Γ at point \vec{r}_0 may be written as

$$v(\vec{r}) = \frac{\Gamma \vec{r} - \vec{r}_0}{4\pi |\vec{r} - \vec{r}_0|^3} \tag{2}$$

To determine the vortex strength on the solid surface of the cantilever plate structure, the surface of the structure may be replaced by a vortex sheet of same geometry as structure and is discretized by a set of linear panels. Individual panels are put on the deformed contour of the surface of structure. Let the surface is evenly divided into N panels and the vortex strength at each vortex point of the panel are $\Gamma_1, \Gamma_2, \Gamma_3 \dots \Gamma_N$ as shown in Figure 1.

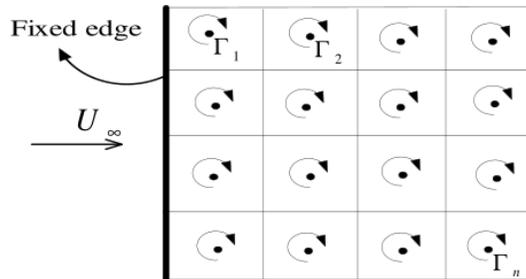


Figure 1. Point vortex at the centroids of each panel on vortex surface

The vortex induced velocity $V_v = (u, v, w)_{i,j}$ at any collocation point i due to a point vortex of strength Γ_j at point j may be determined by the equation

$$V_v(\vec{r}_i) = \frac{\Gamma_j \vec{r}_i - \vec{r}_j}{4\pi |\vec{r}_i - \vec{r}_j|^3} \tag{3}$$

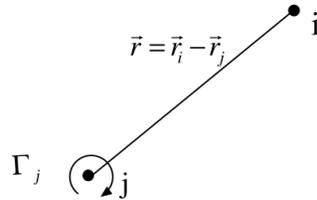


Figure 2. Vortex induced velocity at point i due to a point vortex located at point j

The total sum of vortex induced velocity at any point i in the fluid domain due to vortex $\Gamma_1, \Gamma_2, \Gamma_3 \dots \Gamma_N$ may be written as

$$a_{i1}\Gamma_1 + a_{i2}\Gamma_2 + a_{i3}\Gamma_3 + \dots + a_{iN}\Gamma_N \tag{4}$$

Where, $a_{i,j} = \frac{\vec{r}_i - \vec{r}_j}{4\pi |\vec{r}_i - \vec{r}_j|^3} \quad j = 1, 2, 3, \dots, N$

The induced velocity at collocation point i due to unit strength of vortex at point j may be represent by influence coefficient

$$a_{i,j} = (u, v, w)_{i,j} \cdot \hat{n}_i \tag{5}$$

Where, \hat{n} is the normal of the surface at collocation point i .

2.1 Determination of Influence Matrix

The normal velocity component at each point on the solid surface are the combination of self induced velocity, the kinematic velocity i.e. free stream velocity, vortex induced velocity and it must be zero to satisfy the no penetration condition on the fluid-structure interface at any time. The zero normal flow across the solid surface boundary may be written as

$$(Vv) \cdot \hat{n} + (u_s + U_\infty) \cdot \hat{n} = 0 \tag{6}$$

Where, $U_\infty(t)$ is free stream i.e. kinematic velocity, \dot{u}_s is velocity of structure with reference to coordinate axis fitted with body of the solid, V_v is vortex induced velocity.

Therefore, the normal velocity component due to the all the elements fulfilling the no penetration boundary conditions on the surface at point 1 may be written as

$$a_{11}\Gamma_1 + a_{12}\Gamma_2 + a_{13}\Gamma_3 + \dots + a_{1n}\Gamma_n + \left(U_\infty(t) + \dot{u}_s \right) \cdot \hat{n}_1 = 0 \tag{7}$$

The equation (7) may be written as

$$a_{11}\Gamma_1 + a_{12}\Gamma_2 + a_{13}\Gamma_3 + \dots + a_{1n}\Gamma_n = Rhs_1 \tag{8}$$

Where, $Rhs_1 = -\left(U_\infty(t) + \dot{u}_s \right) \cdot \hat{n}_1$

Specifying the boundary condition for each collocation point the following set of algebraic equation may be obtained for N collocation point and N vortex point on the body surface of the structure

$$\begin{aligned} a_{11}\Gamma_1 + a_{12}\Gamma_2 + a_{13}\Gamma_3 + \dots + a_{1N}\Gamma_N &= Rhs_1 \\ a_{21}\Gamma_1 + a_{22}\Gamma_2 + a_{23}\Gamma_3 + \dots + a_{2N}\Gamma_N &= Rhs_2 \\ \cdot & \cdot \cdot \cdot \cdot \\ a_{N1}\Gamma_1 + a_{N2}\Gamma_2 + a_{N3}\Gamma_3 + \dots + a_{NN}\Gamma_N &= Rhs_N \end{aligned} \tag{9}$$

The equation (9) may be write in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \cdot \\ \Gamma_4 \end{bmatrix} = \begin{bmatrix} Rhs_1 \\ Rhs_2 \\ \cdot \\ Rhs_n \end{bmatrix} \tag{10}$$

The coefficient of the matrix equation (10) $a_{i,j}$ is the velocity component at the collocation point i due to unit strength of point vortex located at point j .

Vortex is formed on the solid surface and washed out from the surface as shown in Figure 3. The vortex formed at the trailing edge is known as wake vortex. For any instant of time the influence of instantaneously wake vortex Γ_w also affect the induced velocity. The latest wake vortex is assumed to be born on the

prolongation of the last panel and to have a longitudinal clearance $0.25U_\infty\Delta t$ from the trailing edge (L Tang et al. 2008). It is assumed that the movement of each individual wake vortex is not affected by the bound vortices or other wake vortices.

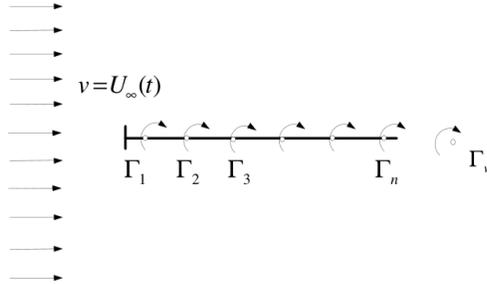


Figure 3. Schematic diagram of formation vortex at the surface of structure and wake vortex at trailing edge

The contribution of a unit strength vortex located at point j towards collocation point i due to all N vortex elements and the latest instantaneously formed wake vortex Γ_w at a given instant of time may be rewritten as

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & a_{1w} \\ a_{21} & a_{22} & \dots & a_{2n} & a_{2w} \\ \cdot & \cdot & \dots & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} & a_{nw} \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \cdot \\ \Gamma_n \\ \Gamma_w \end{bmatrix} = \begin{bmatrix} Rhs_1 \\ Rhs_2 \\ \cdot \\ Rhs_n \\ Rhs_w \end{bmatrix} \quad (11)$$

Where, $Rhs_i = -\left(\dot{u}_s - U_\infty(t) + V_w\right) \cdot \dot{n}$, $U_\infty(t)$ is free stream kinematic velocity, V_w

is velocity component due to the wake vortices (except the latest wake vortex), \dot{u}_s is velocity of structure.

The latest new born vortex may be obtained from the Kelvin condition

$$\frac{d\Gamma}{dt} = \frac{d\Gamma(t)}{dt} + \frac{d\Gamma_w}{dt} = 0 \quad (12)$$

The wake vortex at any time step t may be obtained

$$\Gamma_w(t) = - \left[\Gamma(t) + \sum_{k=1}^{t-1} \Gamma_{w,k} \right] \tag{13}$$

Where, instantaneous circulation $\Gamma(t) = \sum_{j=1}^N \Gamma_j$ is the vortex strength on the surface of the structure.

2.2 Boundary Condition for an Oscillating Cantilever Beam in Oscillating Fluid Flow

A cantilever beam is submerged in an oscillating fluid medium as shown in Figure 4. The beam is performing an oscillation transverse to the flow direction. The strength of vortex on the surface of the structure may be determined by applying the no-penetration fluid -solid interface boundary condition.

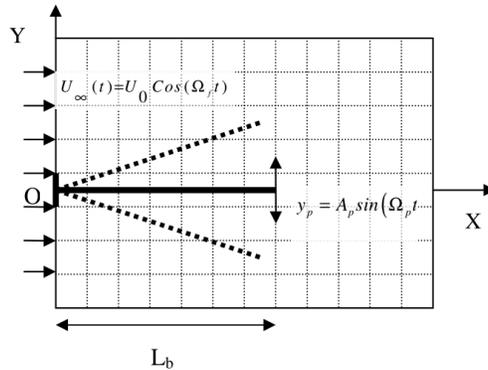


Figure 4. A cantilever beam oscillating periodically in oscillating fluid medium

The no penetration boundary condition for the structural surface may be written as

$$\vec{u}_{vort} \cdot \hat{n} + \vec{U}_\infty \cdot \hat{n} + \dot{y}_p \cdot \hat{n} = 0 \tag{14}$$

Where, $\vec{U}_\infty(t)$ is free stream velocity and it is $\{\vec{U}_\infty\} = \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} \cos(\Omega_f t)$, $\vec{u}_{vort}(t)$ is

vortex induced velocity, \hat{n} is normal at the collocation point on the solid surface at time t, \dot{u}_s is velocity of plate at any time t.

3 Pressure and Lift Force on the Structure Due to Vortex Formation

If the flow field is known then the fluid dynamic pressure on the structure may be computed from the unsteady Bernoulli equation

$$\frac{p_\infty - p}{\rho_f} = \frac{(\nabla\phi)^2}{2} + \frac{\partial\phi}{\partial t} \quad (15)$$

Where, $\nabla\phi = \{u, v, w\}$ velocity of fluid in the domain.

Due to vortex formation on the surface of structure there is pressure difference between lower surface and upper surface of the structure which gives a lift force on the structure.

The pressure difference across the thin vortex surface may be written as

$$\Delta p = p_l - p_u \quad (16)$$

Where, p_l pressure on the lower surface and p_u pressure on the upper surface. Pressure difference between lower surface and upper surface may be written as for unsteady flow (Katz and Plotkin 2001).

$$\Delta p = \rho_f U_\infty \gamma(x) + \rho_f \frac{\partial}{\partial t} \int_0^L \gamma(x) dx \quad (17)$$

Where $\int_0^x \gamma(x, t) dx = \Gamma(x, t)$ is the total vortex strength within the element of length

x.

The normal lift force on the structure due to vortex formation may be written as

$$\Delta L = \int_0^L \Delta p dx = \rho_f \left[U_\infty \Gamma(t) + \frac{\partial}{\partial t} \Gamma(t) \cdot L \right] \quad (18)$$

Lift force at each panel due to circulation $j\Gamma$ may be written as

$$\Delta L_j = \rho_f U_\infty \Gamma_j \tag{19}$$

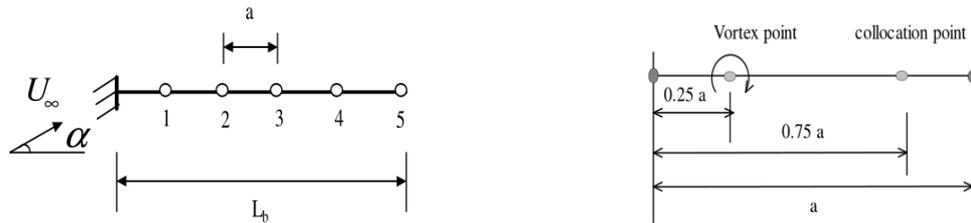
Total lift force on the structure may be written as

$$F_L = \sum_{j=1}^N \Delta L_j \tag{20}$$

4 Numerical Examples

A fluid passes over a flat plate with uniform free stream velocity U_∞ . The centerline of the plate of length L_b is shown in Figure 5. The plate is inclined at a small angle α with the flow direction. The centerline of the flat plate is represented by line (cantilever beam) with five discrete vortices on its surface. The strength of vortices on the surface of the beam is evaluated numerically.

The length of the beam L_b is divided into five number of finite element and length of each element is $a = L_b/5$ as shown in Figure 5 (a).



Cantilever beam discretized into finite element

Vortex point and collocation point within an element

Figure 5. Schematic representation of nodes, vortex point and collocation point on the surface of structural element

The formation of vortices and its strength may be simulated with distribution of point vortices over the surface of the beam. For the analysis the vortex point is placed at position $0.25a$ and collocation point at $0.75a$ of each element as shown in Figure 5 (b) as similar position is chosen by Katz and Plotkin (2001). The strength of vortex on the beam surface is evaluated numerically for a uniform flow velocity passes the structural element and presented in Table 1. The strength of vortex at five selected point on the structural element is evaluated.

The magnitude of the vortex strength is presented in the form $\gamma = \frac{\Gamma}{\pi a U_\infty \sin \alpha}$ in

Table 1 to compare the results with the results of Katz and Plotkin (2001). The strength of vortex at five points is denoted by $\Gamma_1, \Gamma_2, \dots, \Gamma_5$ and corresponding non-dimensional vortex strength is $\gamma_1, \gamma_2, \dots, \gamma_5$.

From Table 1, it is observed that the magnitude of vortex strength at each vortex point is well matched with the results of Katz and Plotkin (2001). The vortex strength decreases slowly in the trailing edge compared to the leading edge.

Table 1. Comparison of vortex strength for uniform flow passes over the beam structure

Vortex Strength	Katz & Plotkin (2001)	Present Analysis
γ_1	2.46092	2.46087
γ_2	1.09374	1.09366
γ_3	0.70314	0.70312
γ_4	0.46876	0.46872
γ_5	0.27344	0.27341

4.1 Numerical Example 1

A fluid with flow velocity $U_\infty(t) = U_0 \sin(\Omega_f t)$ passes over a rigid structural element (Figure 5 (a)) of length (L_b) 0.5m and breadth 0.04m. The structural element is inclined at a small angle $\alpha = 1^\circ$ with the flow direction. The vortex strength and lift force at the surface of the structural element is evaluated for ... at time instant $t=0.2s$.

The strength of vortex at a point $x/L_b=0.2$ on the surface of the structural element is evaluated for sinusoidal fluid flow velocity over the structural element. The variation of vortex strength with fluid flow frequency is presented in Figure 6. It is observed from Figure 6 that the vortex strength at any point varies sinusoidally with fluid flow frequency which is sinusoidal. This shows that the

strength of vortex depends on magnitude of flow velocity and frequency of oscillating fluid flow velocity that are interacting with structural element.

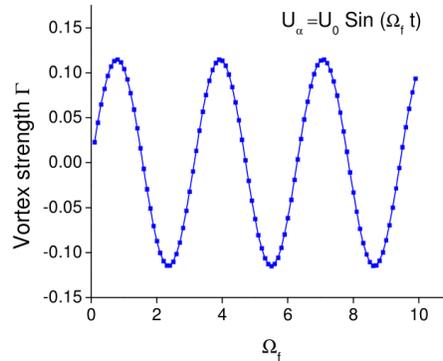


Figure 6. Vortex strength at a point $x/Lb=0.2$ on the surface of structural element for periodic flow $U_{\infty}(t) = U_0 \sin(\Omega_f t)$

For similar conditions, pressure and lift coefficients are evaluated for the structural element and presented in Figure 7. It is observed from Figure 7 that pressure and lift coefficient vary periodically with change of flow frequency. Sudden jump of magnitude of the pressure and lift coefficients are observed at certain interval of flow frequency. The jump may be due to the change of the direction of flow that passes the structural element.

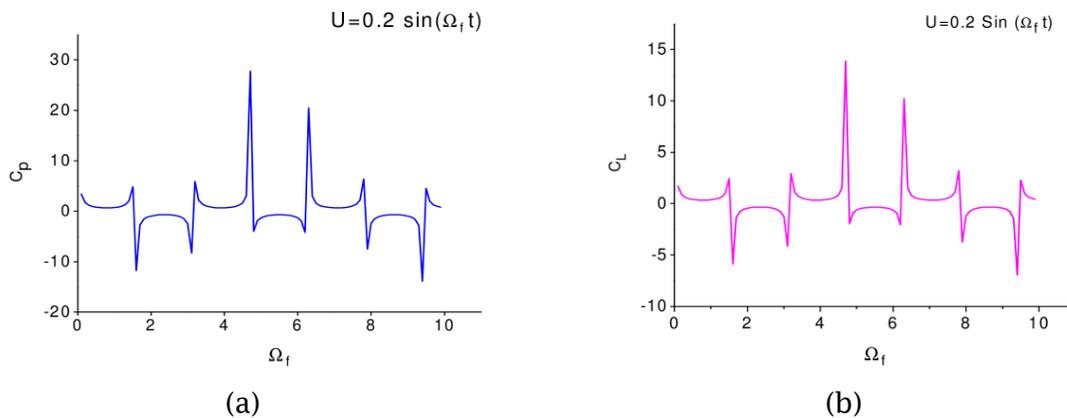


Figure 7. Variation of (a) Pressure coefficient (b) Lift coefficient for sinusoidal motion of fluid over a rigid structural element with different fluid flow frequency

4.2 Numerical Example 2

A structural element (rigid beam) of length (L_b) 0.5m and width 0.04m is considered. The structural element can perform periodical motion in transverse direction in fluid medium as shown in Figure 8. The motion of the structural element is represented by $\theta = \theta_0 \sin(\Omega_b t)$ with $\theta_0 = 0.0044$ and $\Omega_b = 5.2$ rad/s. An oscillating flow $U_\infty(t) = U_0 \cos(\Omega_f t)$ passes over the structural element with $U_0 = 1.0$ m/s and $\Omega_f = 5.2$ rad/s.

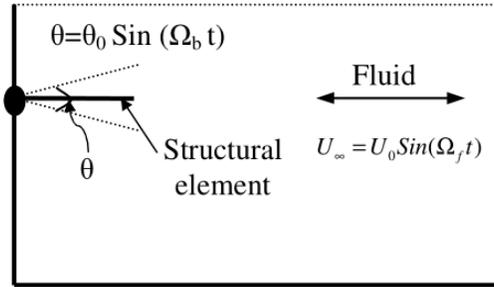


Figure 8. Oscillating rigid structural element in oscillating fluid media

The strength of vortex at a point $x/L_b = 0.2$ on the surface of the beam is evaluated for different frequencies of the structure. The variation of vortex strength is presented in Figure 9 with reduced frequency $\frac{\Omega_b a}{U_\infty}$. Where, a is the distance between two vortices point. It is observed from Figure 9 that the vortex strength at any point on the surface of structural element increases with increase in the reduced frequency $\frac{\Omega_b a}{U_\infty}$.

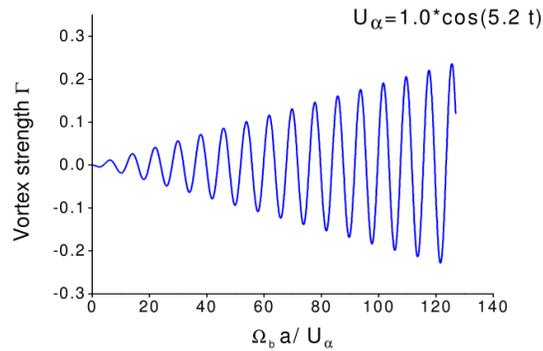


Figure 9. Vortex strength at a point $x/L_b = 0.2$ for sinusoidal flow passes over the transversely oscillated rigid cantilever beam in fluid

For similar conditions, lift and pressure coefficients are evaluated and presented in Figure 10 on the structural element. It is observed from Figures 10(a) and (b) that lift and pressure coefficients depend on the frequency of structural element. With the increase in frequency of the structural element, lift and pressure coefficients increase. Resonance type increment of pressure and lift coefficients are observed with the increase of frequency of structural element.

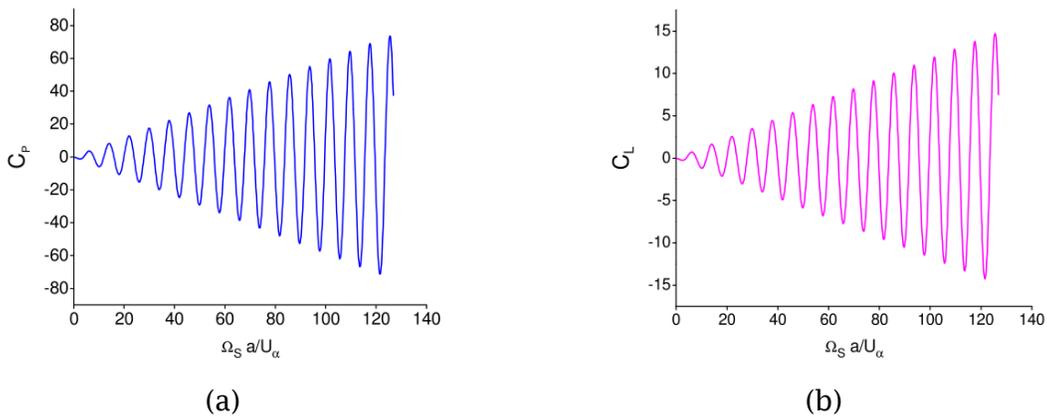


Figure 10. Variation of (a) Pressure coefficient (C_p) (b) Lift coefficient (C_L) for sinusoidal flow over a transversely oscillated rigid beam structure

The variation of vortex strength at a point $x/L_b = 0.2$ on the structural element is evaluated with time for the sinusoidal flow over the structural element and presented in Figure 11. As the vibrational frequency of the structural element

and frequency of fluid flow velocity is kept constant, the strength of vortex at any point on the surface of structure varies periodically.

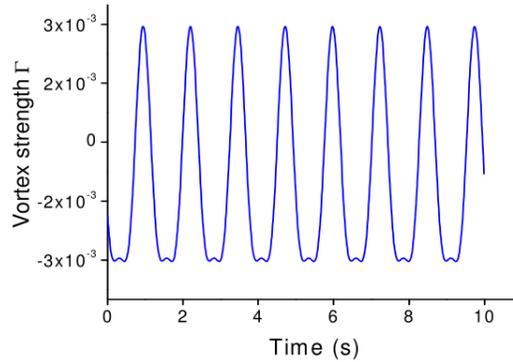


Figure 11. Variation of vortex strength at a point $x/L_b = 0.2$ on structural element with time for sinusoidal flow passes over the structure

The lift and pressure coefficients are evaluated with time and presented in Figure 12. It is observed from Figure 12 that the peak magnitude of lift and pressure coefficients occur at an interval of time.

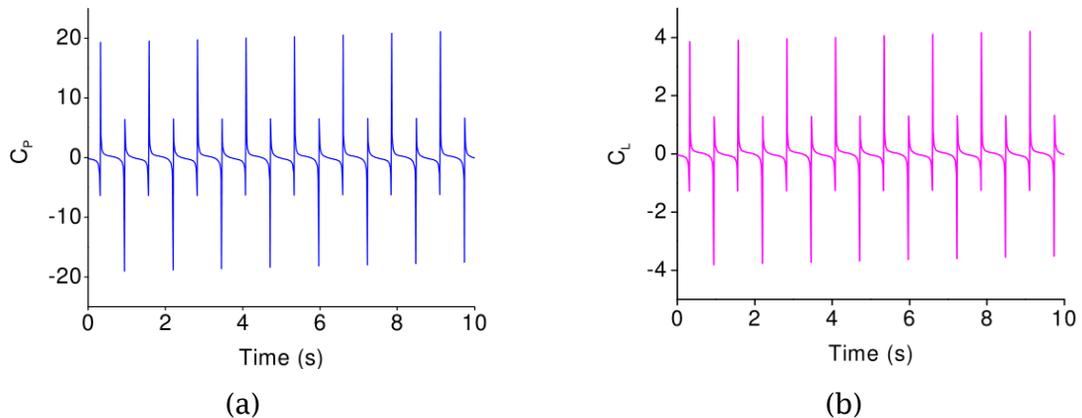


Figure 12. Variation of (a) Pressure coefficient (C_p) (b) Lift coefficient (C_l) with time due to sinusoidal flow passes over a periodically oscillating rigid structural element

5 Conclusion

Vortex is formed on the surface of structural element whenever fluid passes over the structural element. The formation of vortex on the surface of structural

element that is vibrating and interacting with oscillating fluid is simulated. In simulation, it is assumed that the structure vibrates in fluid medium in its vacuum mode. The response of structural element is evaluated by Newmark's time scheme. The deformed shape of structural element at any instant of time is replaced by an imaginary vortex sheet. The strength of vortices at any instant of time is evaluated by velocity compatibility at the interface of fluid and structure. A number of point vortices are distributed on the surface of vortex sheet. Mathematically, the point vortices are singular point on the surface of deformed shape of the structural element. Green's function of Laplace equation is applied for finding the vortex strength. The strength of vortex depends on fluid velocity and its frequency. The lift and pressure forces on the structural element due to vortex formation is evaluated and it is observed that pressure and lift forces depends on the velocity of structure and flow velocity around the structural element.

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